The aims of this workshop are:

-To understand critical path analysis and the types of problems that are solved with critical path analysis.

-To understand how maths and operational research are used in real life.

OPERATIONAL RESEARCH IN EDUCATION

Critical Path Analysis

Critical path analysis involves planning the order of activities in a project.

It's helpful when activities take different amounts of time and some activities depend on others.

It determines the shortest possible time needed to complete the project.

It's something you probably do all the time without realising it…

Optional: ask the class if they can think of any examples of projects where critical path analysis would be helpful.

OPERATIONAL RESEARCH IN EDUCATION

Making a cup of tea

Critical path analysis is planning the order of activities in a project.

What order do you do things in when you make a cup of tea? Why?

Critical path analysis is helpful when some activities depend on others.

You can't let your tea brew before you've boiled the kettle, therefore the brewing is dependent on the earlier step.

Ask the students to think about how they make a cup of tea, and come up with a short list of steps (it may help to write these on the board). Ask students why the steps are in that order. We want them to realise that you can't let your tea brew/add hot water to your cup etc. without boiling the kettle first.

(Bottom two sentences appear on click.)

This is a good example of a **dependency.** Dependencies in critical path analysis indicate which activities must be completed before other activities can start.

OPERATIONAL RESEARCH IN EDUCATION

Getting ready for school/college

Critical path analysis determines the shortest possible time needed to complete the project.

You might have used critical path analysis to determine the shortest time you needed to get ready this morning, and therefore what the latest time you can wake up and still be on time is.

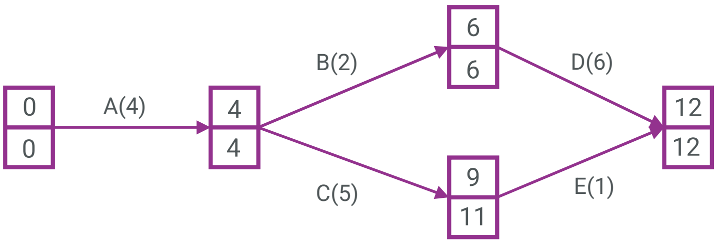
Critical path analysis is about finding out the shortest time you need to complete a set of activities. Students know what time they need to leave the house to get to school/college on time, and roughly how long they take to get ready and eat breakfast. Therefore they can work backwards to find out what time they need to set their alarm for, maximising their sleep!

You can optionally bring dependencies into this example - students need to get dressed before they leave the house, and leave the house before they can catch the bus etc.

OPERATIONAL RESEARCH IN EDUCATION

Critical Path Analysis Diagrams

For more complex, or less intuitive projects, a critical path diagram is helpful.



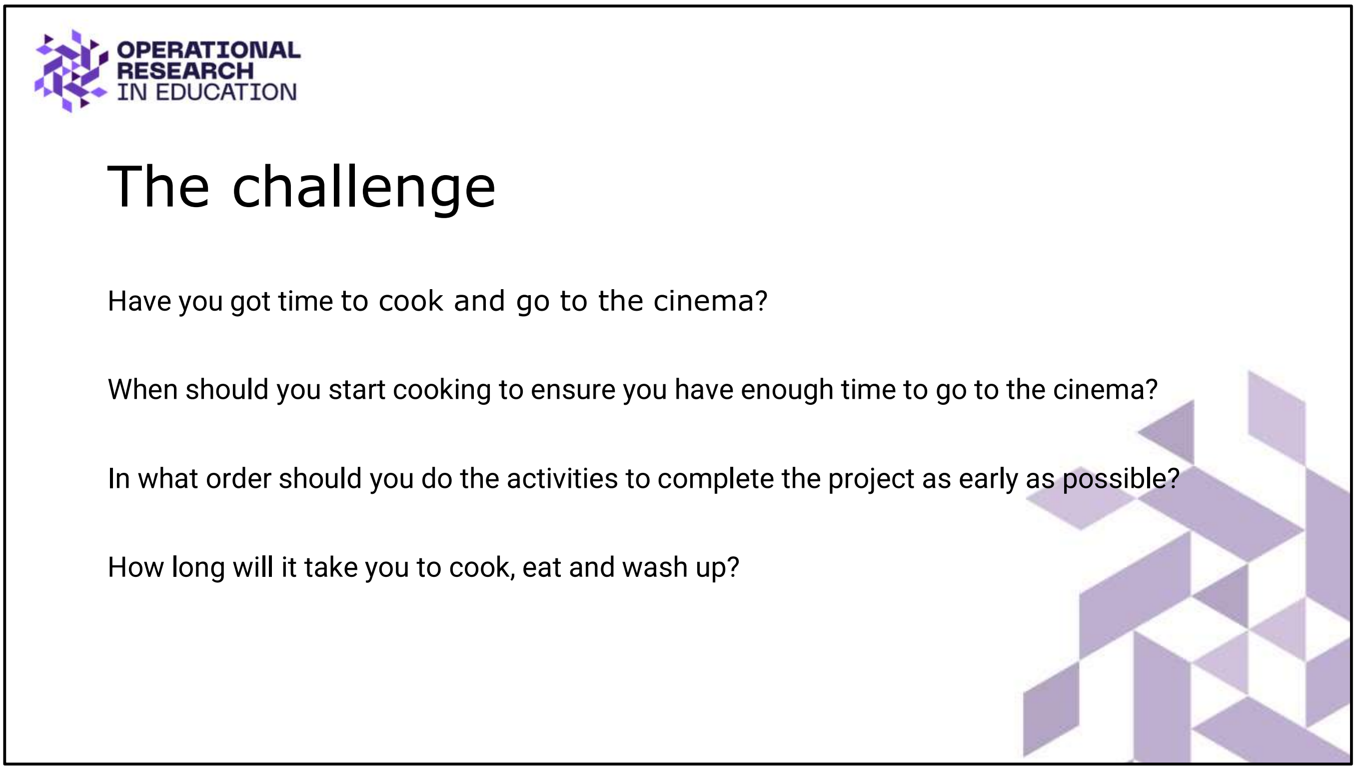
If it’s a business project (such as relocating a store, or designing/ manufacturing/ selling a new product), critical path analysis and diagrams are invaluable. The slide shows an example of a simple critical path diagram. We’ll go through how to create and label a critical path diagram next…

Cooking Bolognese

You want to cook spaghetti Bolognese for tea tonight, but your friends want to go to the cinema with you.

Your task is to find the quickest way to cook spaghetti Bolognese, so that you can tell your friends when you can go to the cinema.

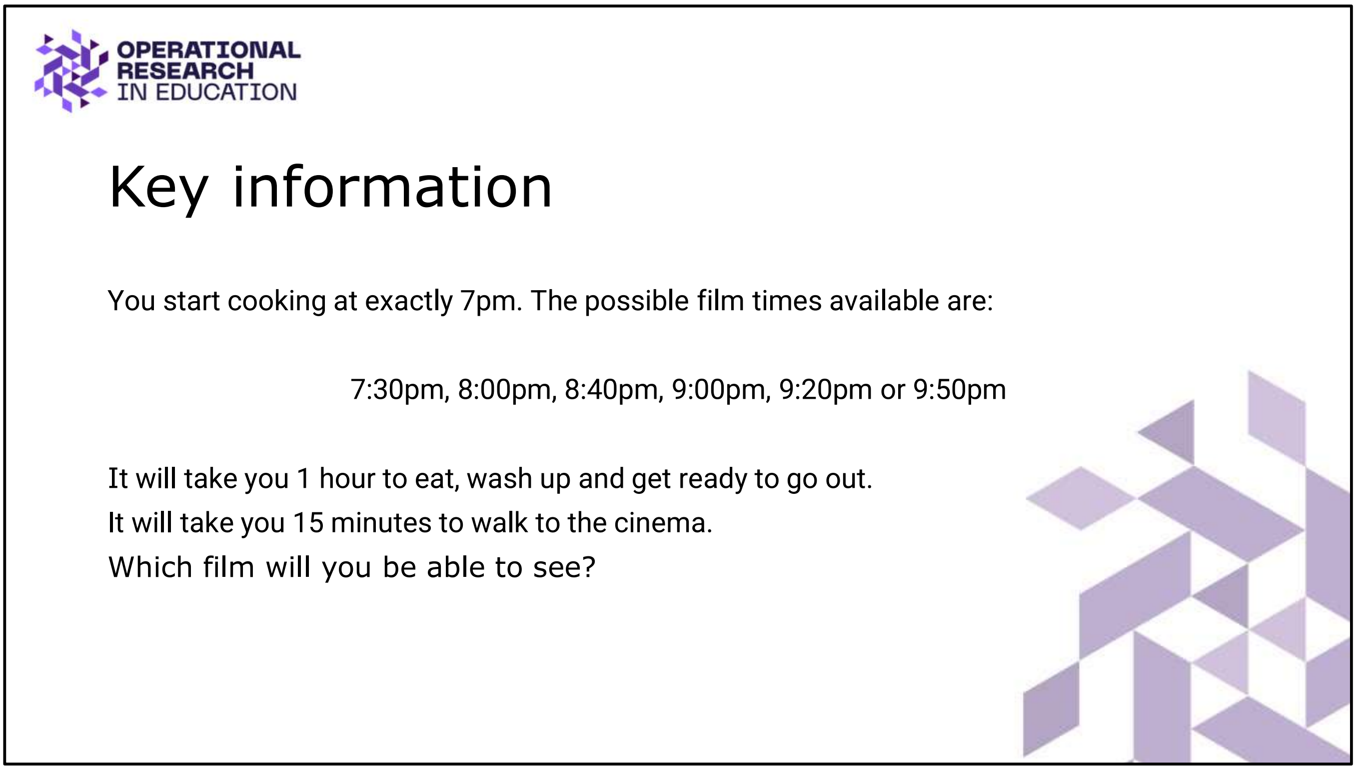
Introduction to the cooking Bolognese worked example



Why use critical path analysis to solve this problem? It helps you to answer all of these questions and make sure your ‘project’ (in this case your evening) runs smoothly.

If students ask why not buy food on the way to or at the cinema, perhaps in this scenario they are uni students on a tight budget and can’t afford to eat out and go to the cinema.

If students ask why not cook a quicker dinner, maybe they’re trying to cut down on food waste and need to use ingredients up tonight before they are out of date.



If students query why they start cooking at 7pm, not sooner, maybe it was a last minute plan and they didn't know they were going out until 7, maybe they didn't get home until 7pm, etc.

OPERATIONAL RESEARCH IN EDUCATION

The recipe

You have managed to find a recipe online outlining the different tasks you need to complete to cook spaghetti Bolognese.

The recipe has been written on 7 different cards and mixed up.

Put the cards back in order and create a timeline.

Student worksheet question 1

Hopefully when the students try to create a timeline for the recipe there will be some discussion – should they start with cooking the mince/quorn as it takes the longest, or cook the pasta and then the Bolognese? You could argue you should chop the vegetables first to get all of the prep work done at the beginning. There’s no wrong answer, but our goal here is to find the best answer, the quickest way to cook dinner.

OPERATIONAL RESEARCH IN EDUCATION

The recipe

Here are the tasks and their durations:

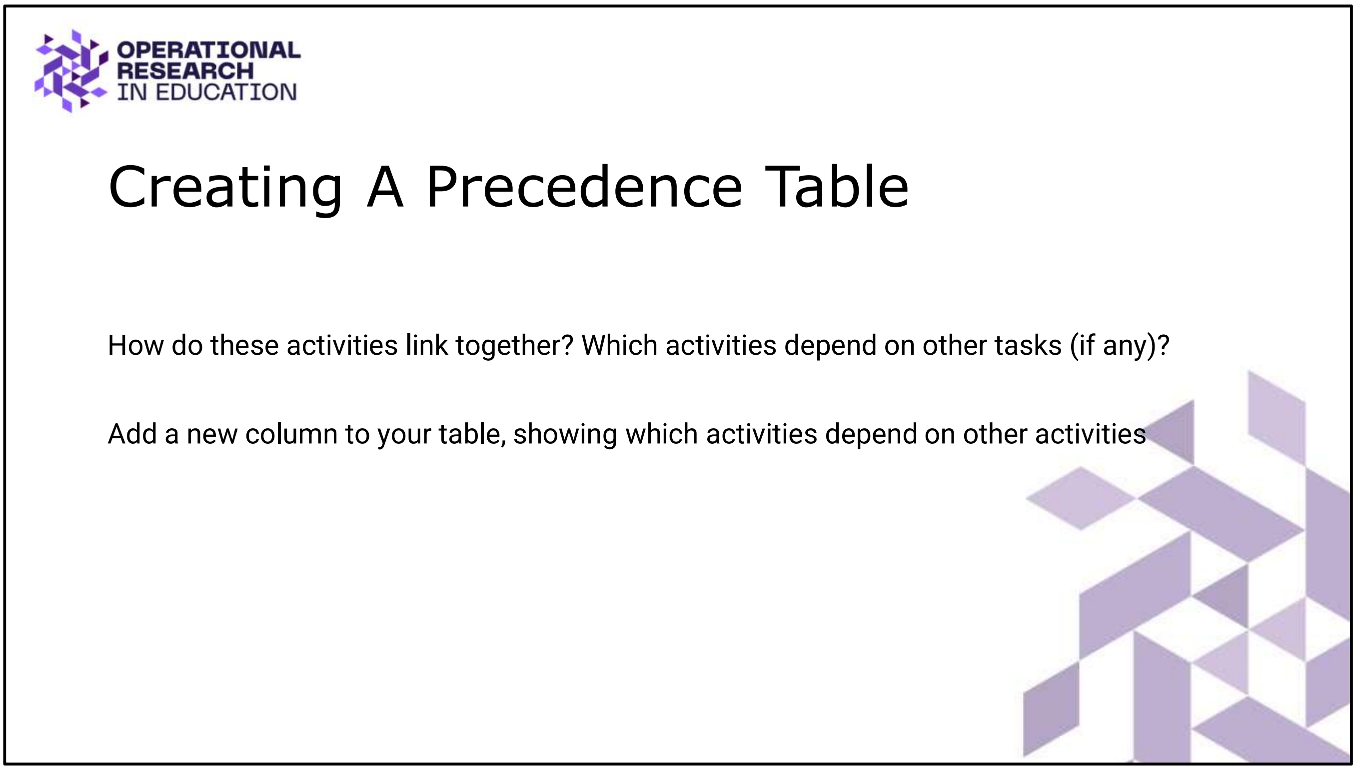
Activity	Label	Duration (minutes)
Boil water	A	5
Add pasta to water and cook	B	10
Chop up vegetables	C	7
Cook mince/quorn	D	30
Add vegetables and tomato puree to mince/quorn and stir	E	3
Drain pasta	F	2
Mix Bolognese and pasta together and serve	G	2

Student worksheet question 2

Ask students to fill out the activity and duration columns in the table on their worksheets so that they match the slide.

We’ve given the tasks random labels to make creating a diagram easier later, as well as the table on the next slide.

The label A doesn’t necessarily mean you’d start with boiling your water.



Student worksheet question 3

Ask students to fill out the ‘depends on’ column in their tables. This will turn their tables into ‘precedence tables’.

Precedence Table

Your precedence table should look like this:

Activity	Label	Duration (minutes)	Depends on
Boil water	A	5	-
Add pasta to water and cook	B	10	A
Chop up vegetables	C	7	-
Cook mince/quorn	D	30	-
Add vegetables and tomato puree to mince/quorn and stir	E	3	C,D
Drain pasta	F	2	B
Mix Bolognese and pasta together and serve	G	2	E,F

This is called 'a precedence table' and helps to organise the activities and show the constraints.

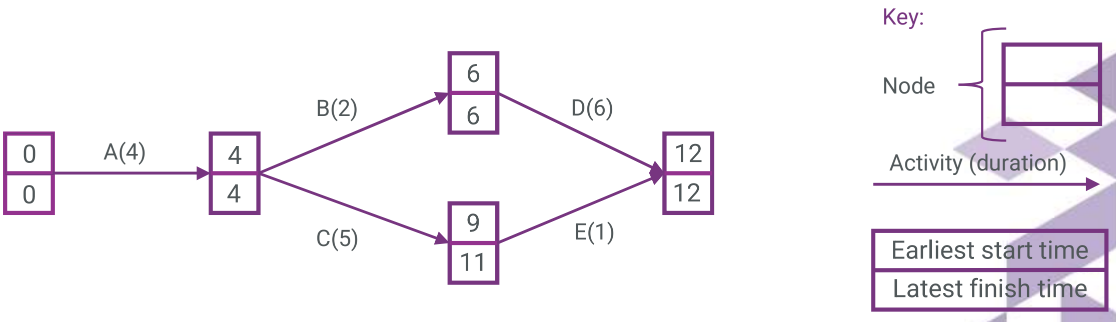
Notice that the dependencies only refer to the activities immediately before the activity in question.

For example, when you make a cup of tea, brewing the tea cannot happen before you have boiled the kettle!

OPERATIONAL RESEARCH IN EDUCATION

Activity Networks

Activity networks are diagrams that show links between activities.

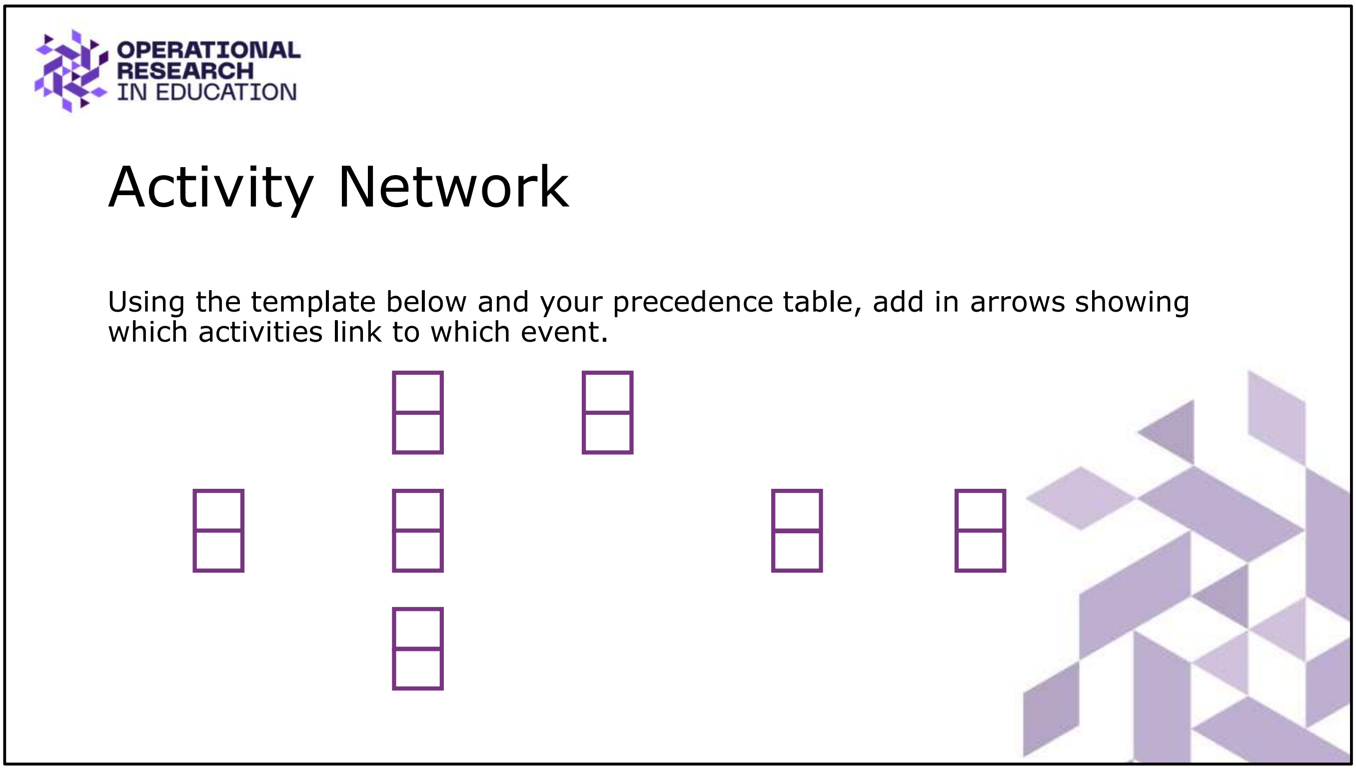


When we draw an activity network, or a critical path diagram, the activity or task takes place on the arrow, or on the arc. The duration of the activity is in the brackets after the label.

The nodes, the boxes between arrows, represent events. An event is not a task, it is the time when one task is finished and the next can begin. The earliest start time and latest finish time correspond to the events, rather than the activities. These will be covered in more detail later on.

Events will link activities that have dependencies, for example here B and C depend on A as they start after the event that represents A being completed.

This can seem counter-intuitive so spend a while ensuring the information has sunk in if needed.



Student worksheet question 4

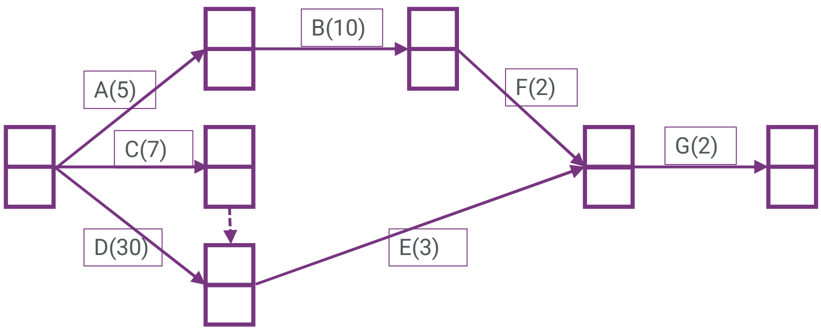
Ask students to try adding arrows to the diagram, showing which activities link to which event. Remind them that the arrows represent activities such as cooking their pasta.

They may want to work in pencil rather than pen!

OPERATIONAL RESEARCH IN EDUCATION

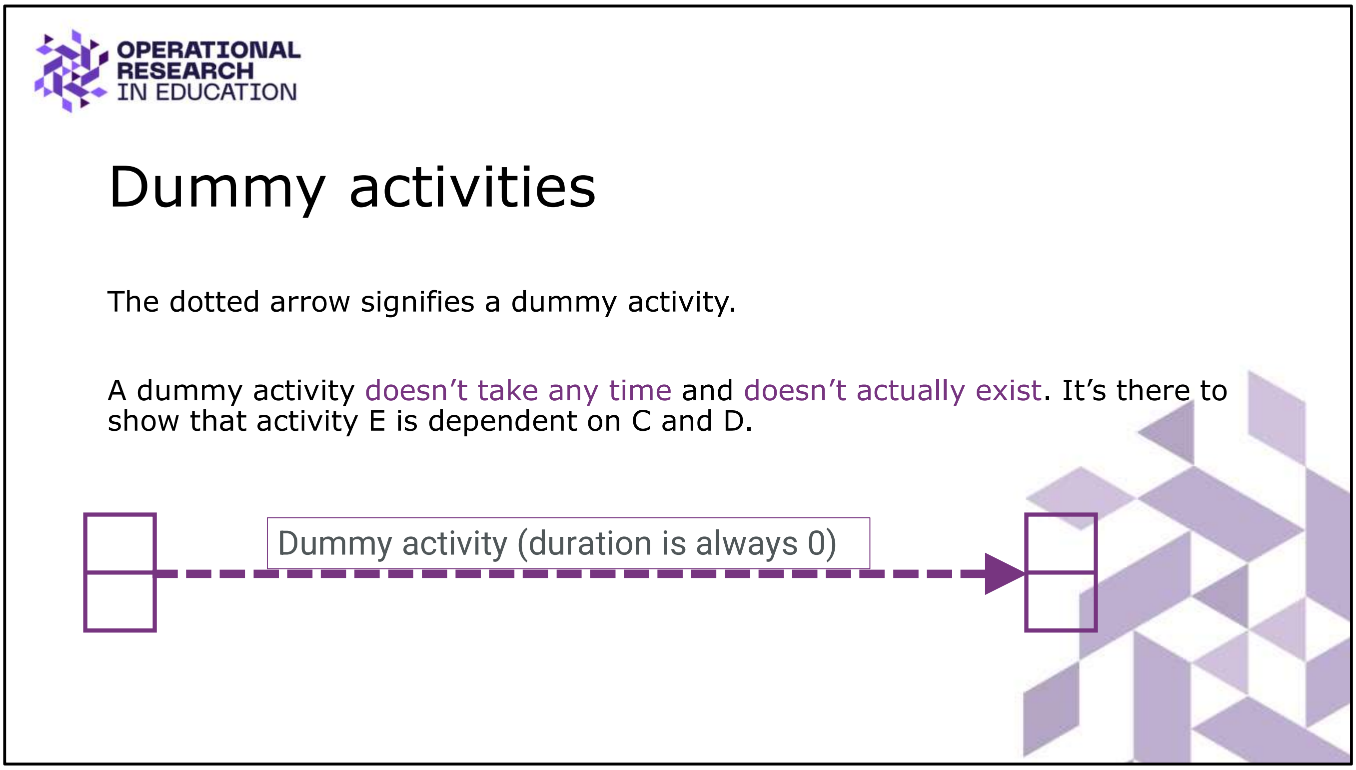
Activity Network

Using the template below and your precedence table, add in arrows showing which activities link to which event.



Talk through the diagram. Activity A leads into the node Activity B comes out of because B is dependent on/preceded by A. Activities E and F lead into the node G comes out of because G is dependent on both E and F.

The dotted line in between C and D is a dummy activity (more on the next slide). It is there to signify that E is dependent on C and D, but the dummy activity is instantly complete, with a duration of 0. Dummy activity best practice is that it runs from the activity that finishes first (here C) to the activity that finishes second (here D).



Students may find it easier to write (0) near their dummy activity. This isn't needed to complete the diagram but isn't incorrect. Students can use whichever method they prefer.

Now that we have the activities in the right place, we can move on to filling out the nodes with the Earliest Start Times and Latest Finish Times

OPERATIONAL RESEARCH IN EDUCATION

Earliest start time & latest finish time

The earliest start time (EST) is the earliest possible time an activity can begin without reducing the duration of any of the preceding activities.

The latest finish time (LFT) is the latest possible time an activity can end without delaying the start of any of the following activities.

These definitions are important as we are about to calculate the EST and LFT for our network diagram.

OPERATIONAL RESEARCH IN EDUCATION

Earliest Start Time (EST)

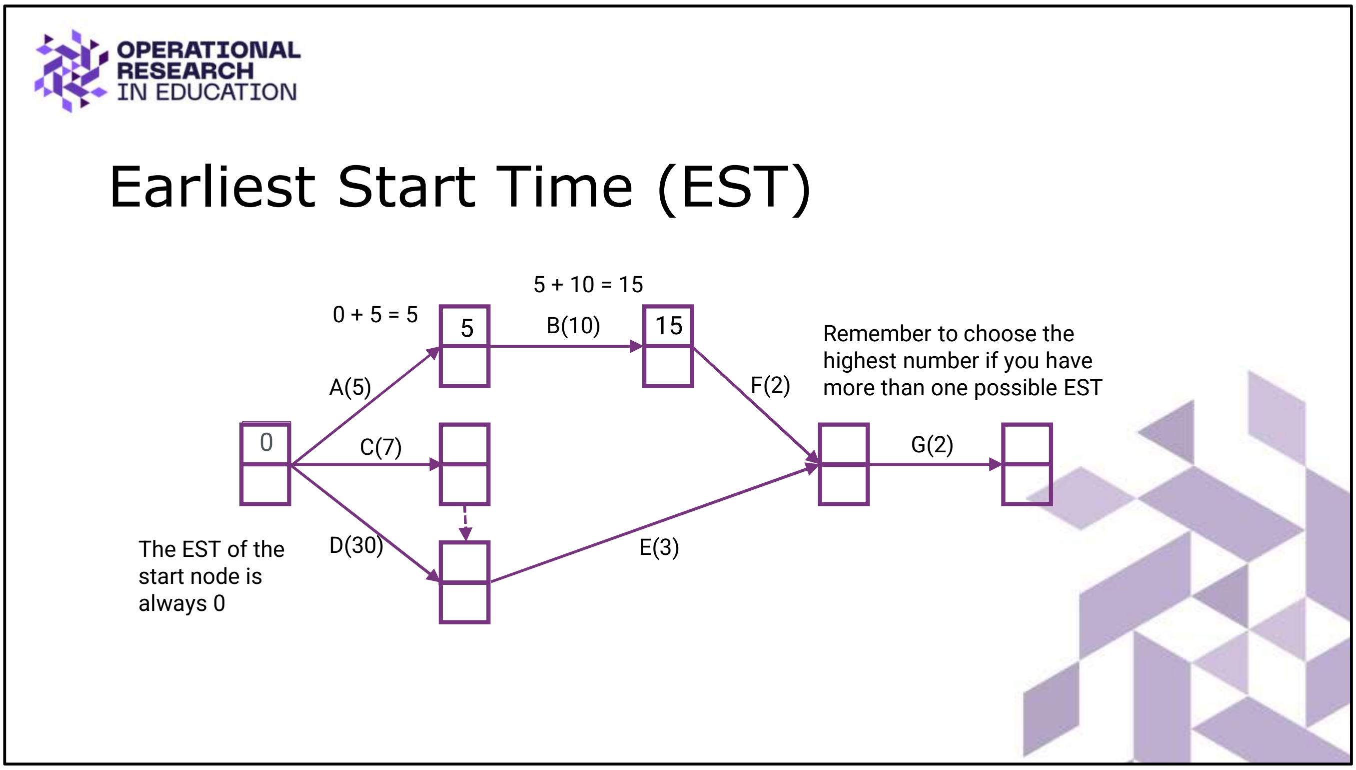
Work forwards, from the start node to the end node, to calculate EST.

EST = Earliest Start Time of the previous activity + Duration of the previous activity

If there are two or more previous activities, choose the highest EST.

It’s important to choose the highest EST if you have more than one option. Can any of the students work out why?

If you choose a lower EST you’ll suggest a task can start before all of the tasks it is dependent on have finished. For example, C depends on A (duration 5) and B (duration 10). If you start C at 5, B hasn’t finished and your timing will be wrong. In our cup of tea example, this would be like trying to pour the hot water or brew the tea before the kettle had finished boiling.

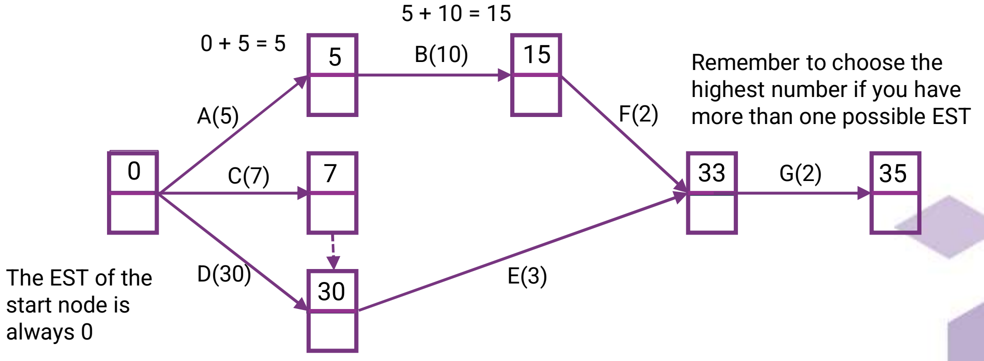


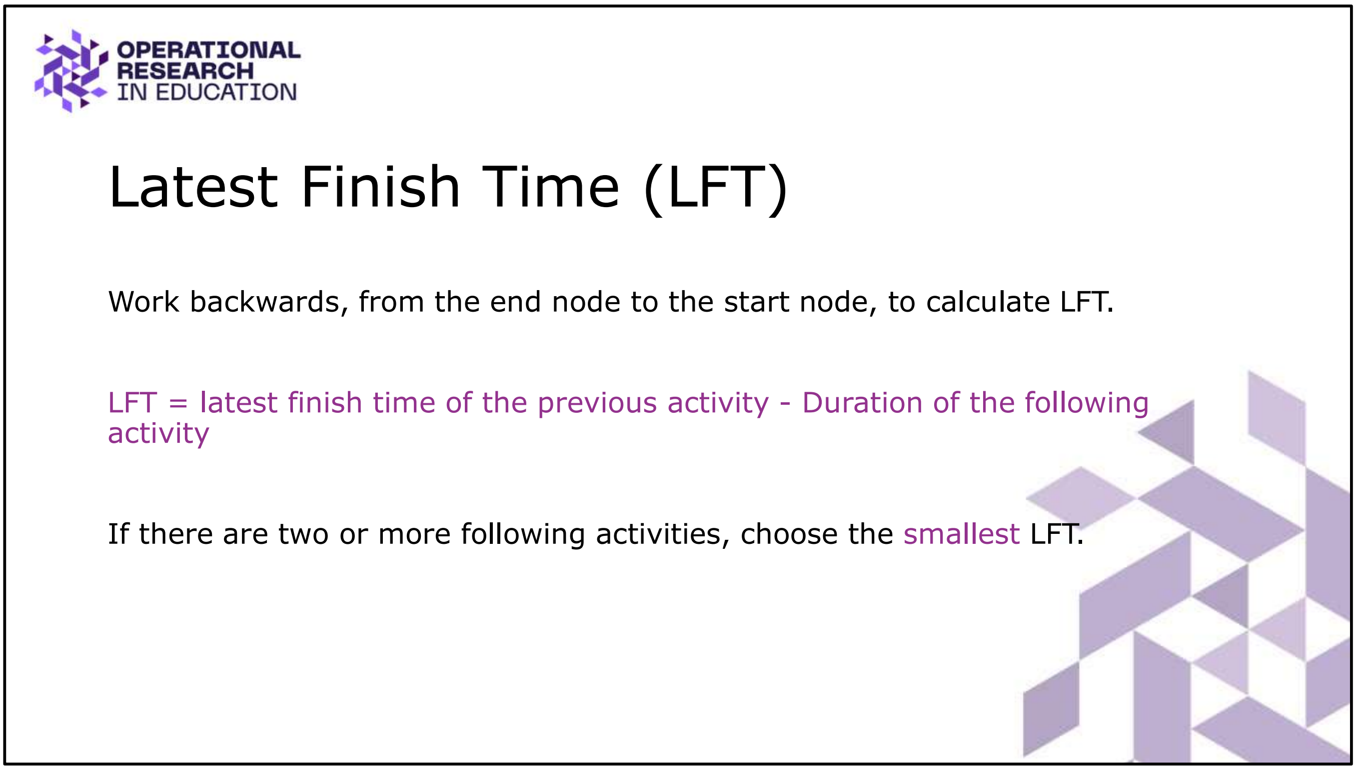
EST = Earliest Start Time of the previous activity + Duration of the previous activity.

Talk through the examples with the students and encourage them to fill out the rest of the diagram independently. Click through for answers on the following slide.

Student worksheet question 5

Earliest Start Time (EST)

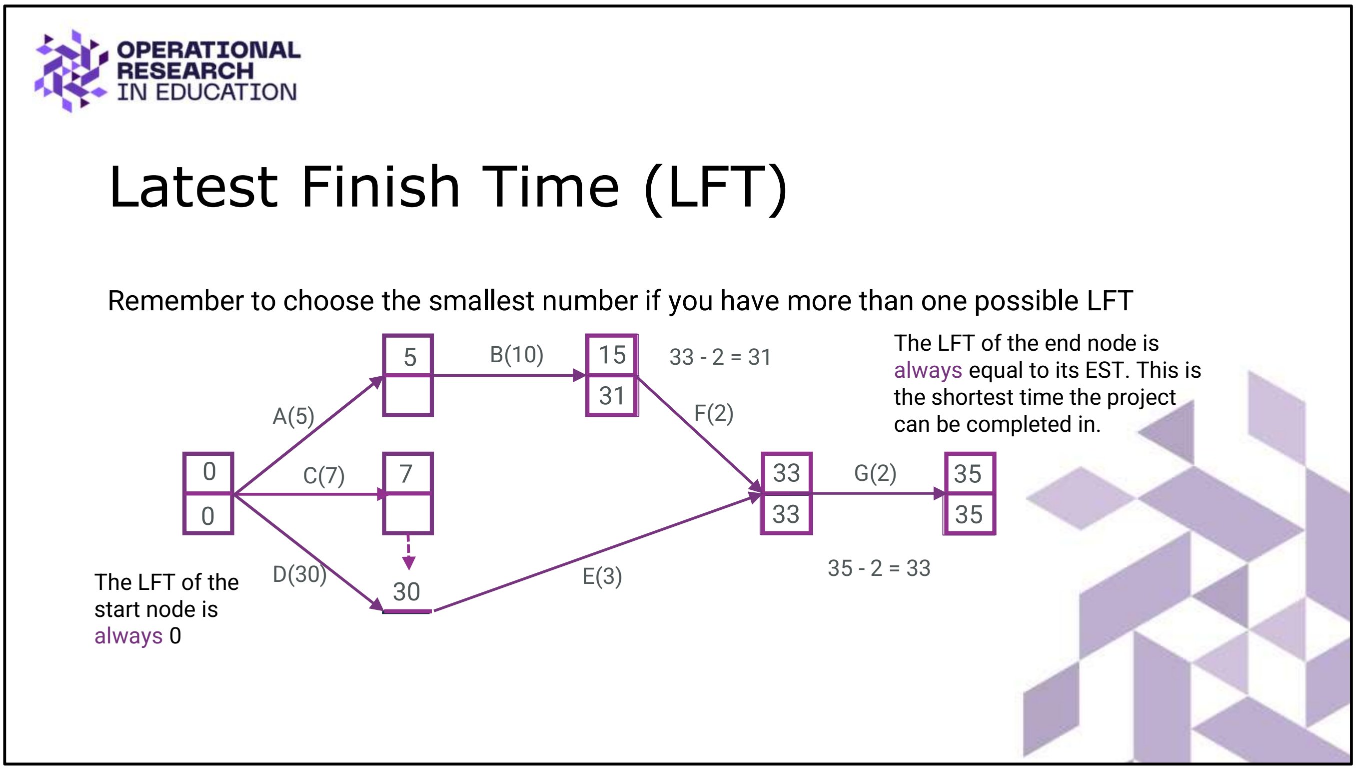




The LFT is the latest time an activity can finish *without making any other activities late*.

It's important to choose the smallest LFT if you have more than one option. Can any of the students work out why?

If you choose a higher LFT you might slow down subsequent tasks in the project – remember our overall goal is to finish as quickly as possible.



For LFT we start from the end node and work backwards to the start node.

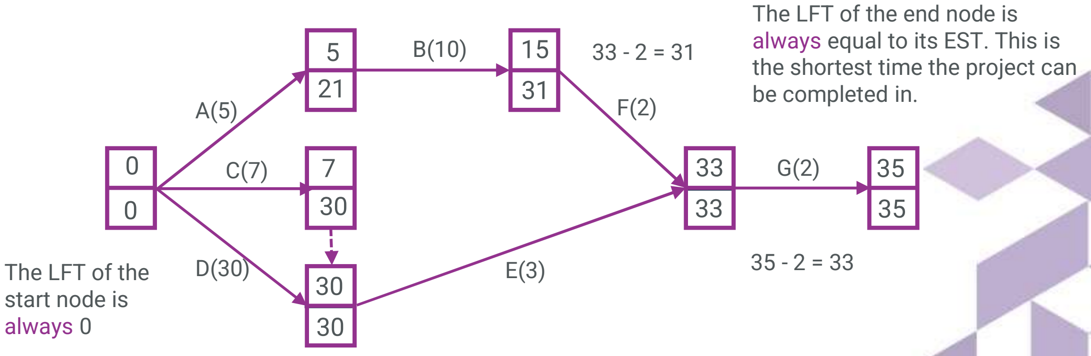
LFT = Latest Finish Time at the end of the following activity – Duration of the following activity

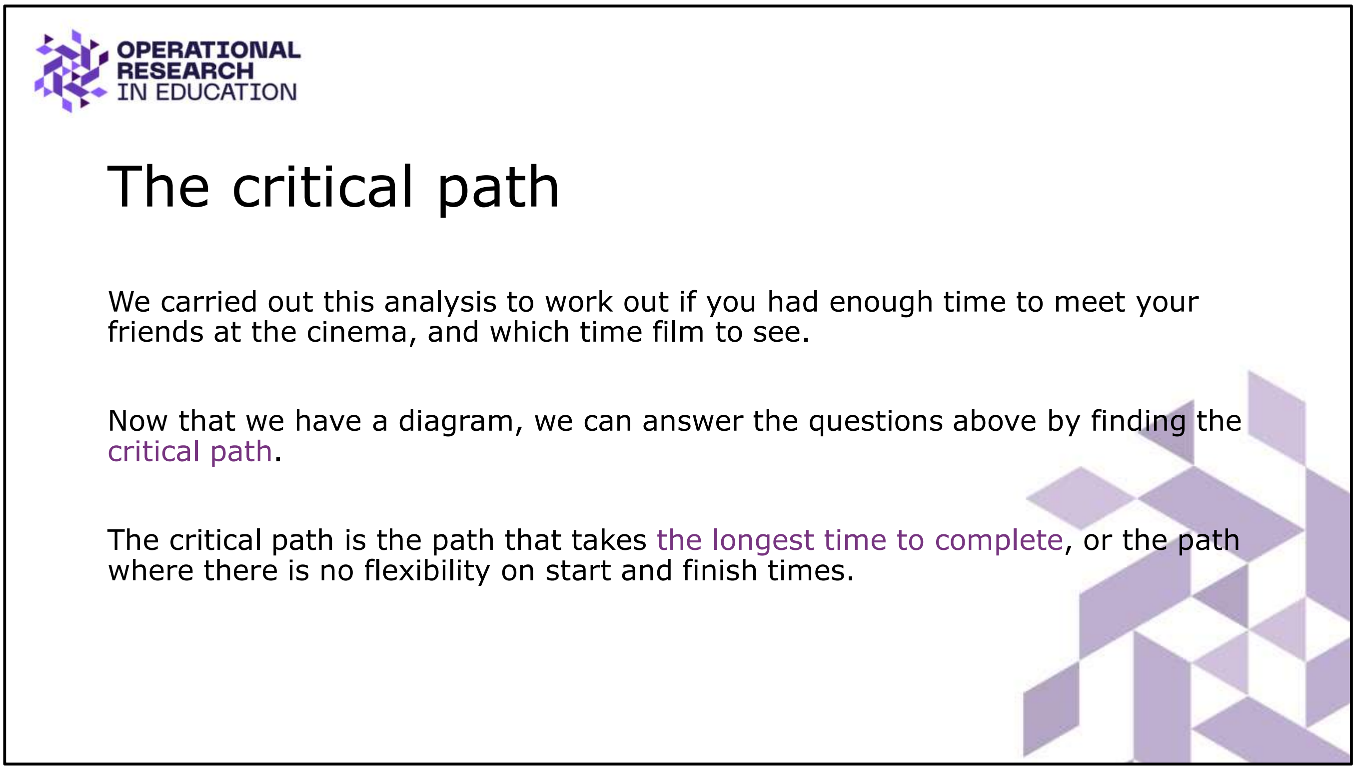
Talk through the examples with the students and encourage them to fill out the rest of the diagram independently. Click through for answers on the following slide.

Student worksheet question 6

Latest Finish Time (LFT)

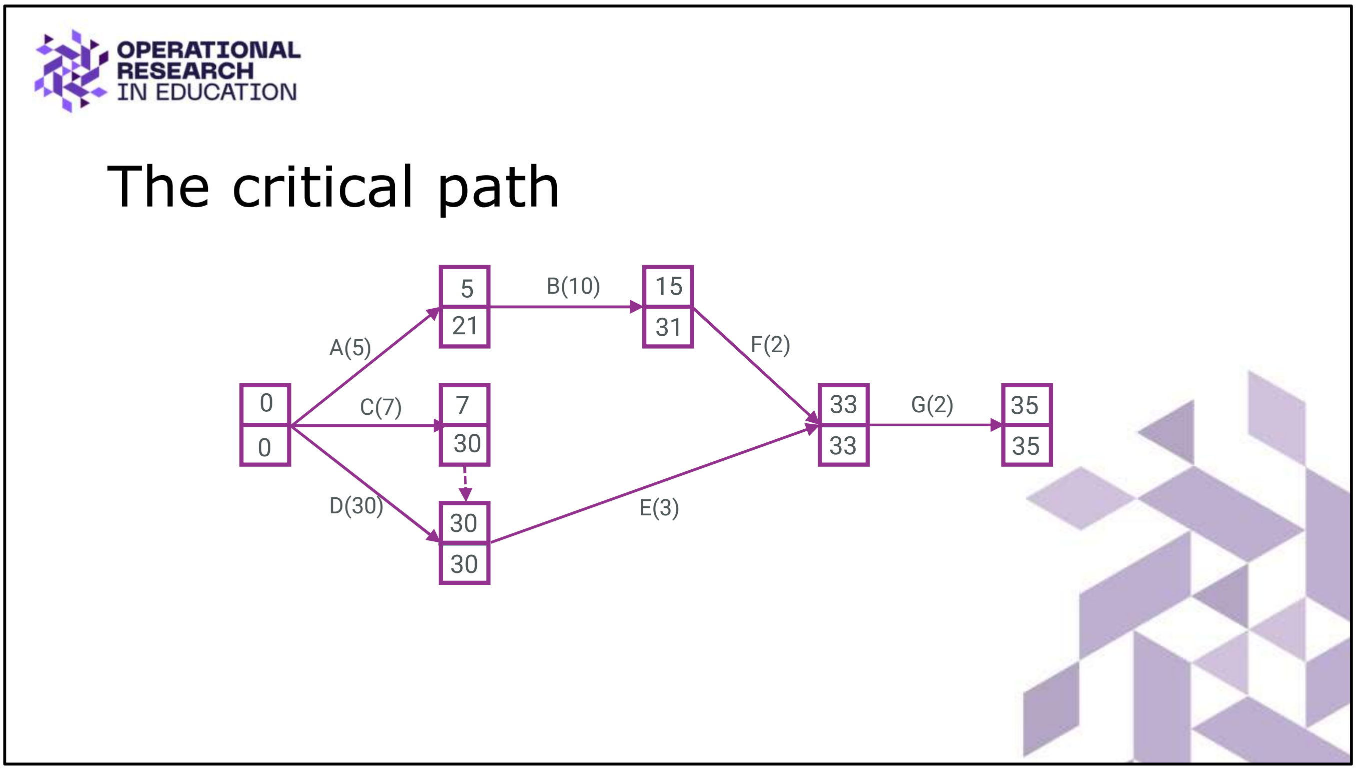
Remember to choose the smallest number if you have more than one possible LFT.





The critical path is the path that contains no float time. If any of the activities on the path start late, they will also finish late and therefore delay the project. This is what makes them critical.

Float time is the amount of time you have to complete a task (LFT – EST) minus the duration of the task. Example on next slide.



Activity C has a float of 23.

(LFT (30) – EST (0)) – duration (7) = 23.

What do students think the float on B is? Answer: ((31-5) – 10) = 16

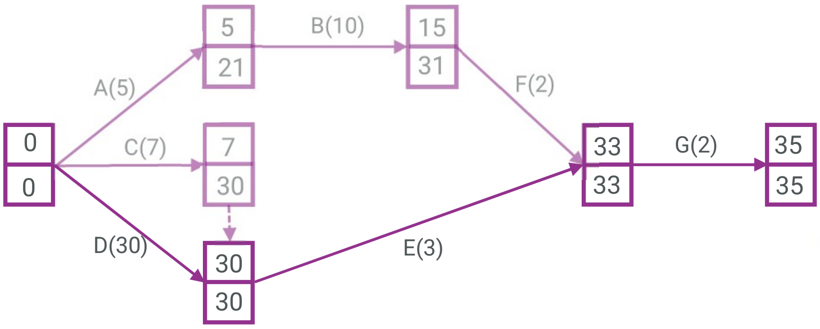
Activities on the critical path have a float time of 0.

Student worksheet question 7

What do students think the critical path is? Answer on next slide.

OPERATIONAL RESEARCH IN EDUCATION

The critical path

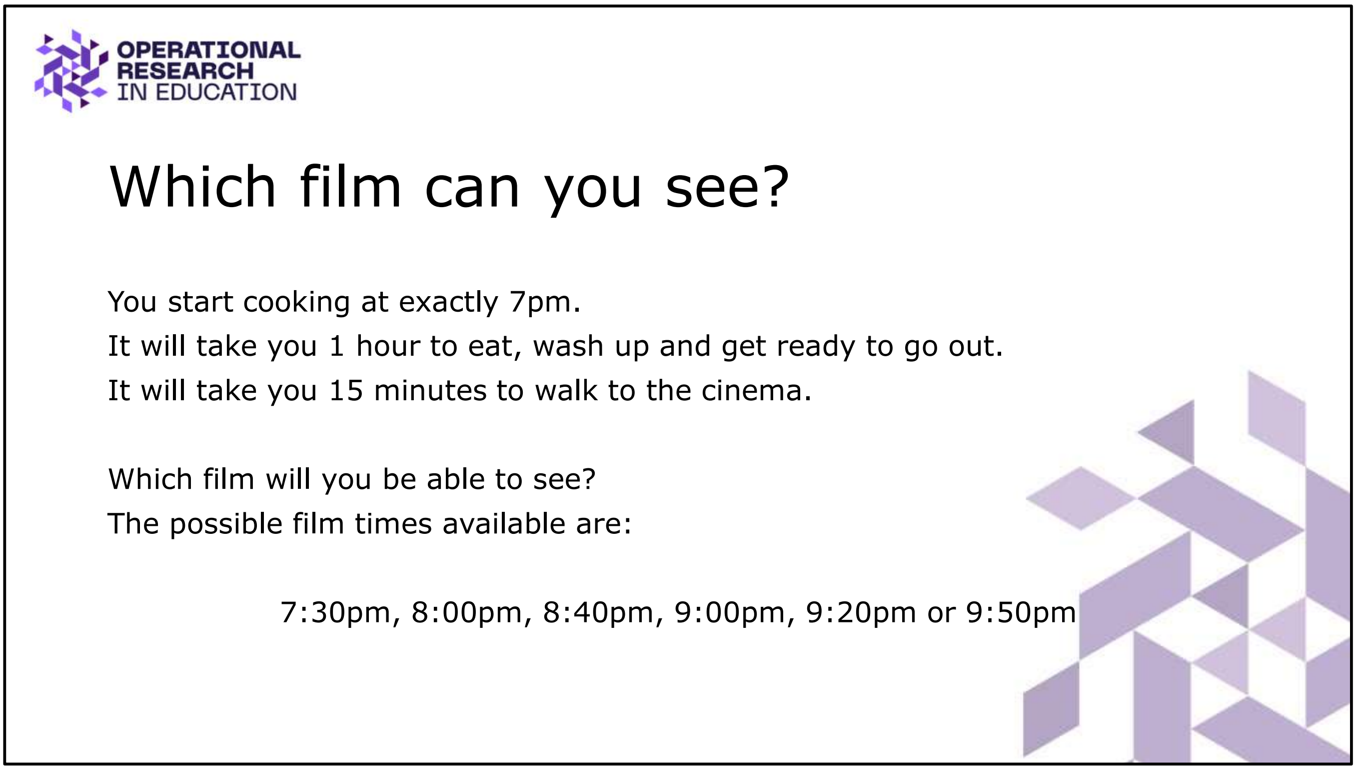


What does this tell us?

The critical path is D-E-G. The shortest time the meal can be cooked in is 35 minutes.

A quick way to identify the critical path is that the EST and LFT times will be the same, because there is no float time.

What does this mean for our cinema trip?



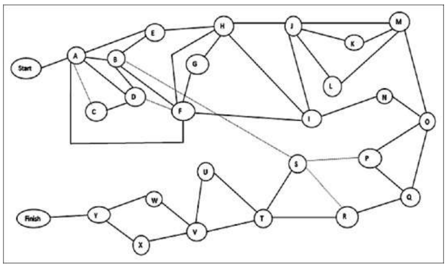
Student worksheet question 8

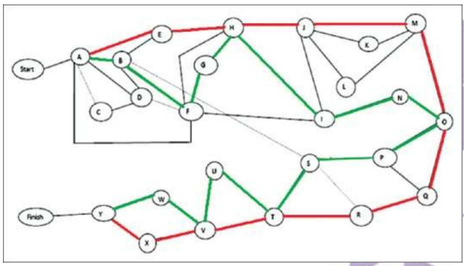
The 35 minute time to cook dinner, plus the 60 minute eating/getting ready time and 15 minute journey mean we need 110 minutes from the beginning of cooking to arrive at the cinema. If we start cooking at 7pm the earliest we could get to the cinema for is 8:50pm, for the 9pm film.

OPERATIONAL RESEARCH IN EDUCATION

Critical Path Analysis

Critical path analysis has a wide variety of uses including in engineering, construction, aerospace and product development.





https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4759980/

Critical path analysis is a widely used project management tool. The advantages to efficient project management will vary from industry to industry, but in general having a more accurate idea of how long a project will take will help with business planning, budget estimates and staff or raw material requirements.

For example, if a company can predict whether or not its new gadget or toy will be developed in time for the busy Christmas shopping period is useful as that could have a big impact on sales and therefore profit.

Home renovations and housebuilding are also good examples. You probably don’t want to put down new carpet before you’ve painted your walls, or install windows before you’ve built your walls.

The images are from a 2016 article on CPA in planning clinical drug trials. There they had 25 different activities labelled A – Y. The red path in the second image is the critical path. The advantage to completing a drug trial quickly is clear – (if it is a success) patients can access the new medicine more quickly.

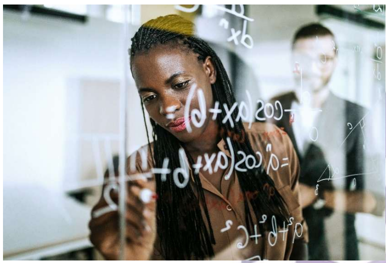
OPERATIONAL RESEARCH IN EDUCATION

Operational Research

Operational research (OR) is the application of mathematical methods and advanced analysis to improve decision-making

Or:

'The science of better decision-making'



Ask the students if they have heard of operational research. Often not many people have. (Text appears on click/moving forward).

The answer on the slide can also be stated as "OR involves using maths to solve problems or make better decisions". It is a broad, slightly vague answer – that's because OR has lots of practical applications!

OR is used today by many businesses – shops, airlines, architects, hospitals, local government and central government (think Ministry of Defence, HMRC etc.)

There are some in depth examples of OR on the following slides. Feel free to include your own.

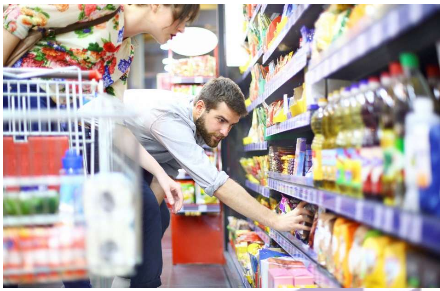
OPERATIONAL RESEARCH IN EDUCATION

OR in detail - Supermarkets

Understanding people’s buying patterns

Determining the number of staff needed on checkout and when

Calculating order quantities and delivery times



Please feel free to paraphrase the below:

Supermarkets use teams of OR professionals to solve problems and make decisions, such as understanding consumer buying patterns, deciding how many staff they should allocate to a shift and calculating the optimal quantity and delivery times of their products.

Supermarket loyalty cards, like a Tesco’s Clubcard, are a great example of OR in action. Loyalty cards let supermarkets track what their customers are buying, creating huge amounts of data for operational researchers to work with. They can use statistics to search for patterns in the data, attempting to predict how customers will behave in the future.

For example, the data might show that people buy lots of milk on a Saturday, in which case the supermarket would know to stock up on Friday evening. It might also show that lots of people shop at certain times, or on a particular day, so the store managers would know to have more staff members working at that time.

Most supermarkets also incorporate weather forecasting data, obtained from weather stations near each of their stores to optimise this further by making sure they have extra BBQ food in towns that are expecting sunny weekends.

It’s easy to see what a big impact OR has on making the right decisions for supermarkets – helping them keep customers happy and make profits!

OPERATIONAL RESEARCH IN EDUCATION

OR in detail - Airlines

Forecasting where people want to go and when

Setting the ticket prices

Simulating boarding the plane

Please feel free to paraphrase the below:

Operational researchers at places like British Airways are involved in a lot of decision-making.

When you book a holiday, OR has been used to decide where an airline will fly to and how much they charge you for your ticket, using customer buying patterns and forecasting to predict demand.

When you arrive at the airport, OR has been used to minimise queueing times, and simulations are used to model the flow of passengers through the terminal to ensure staff members and equipment are in the right places at the right time.

When you board the plane, OR has helped choose a boarding strategy and ensure your plane leaves on time. OR is even used to forecast how many passengers are likely to cancel their holiday!

Just like supermarkets, airlines rely heavily on OR to make better, more informed decisions that result in better outcomes for their business.

OPERATIONAL RESEARCH IN EDUCATION

OR in detail - Healthcare

Moving people through waiting lists as quickly as possible

Using hospital resources as efficiently as possible

Increasing the number of transplant operations

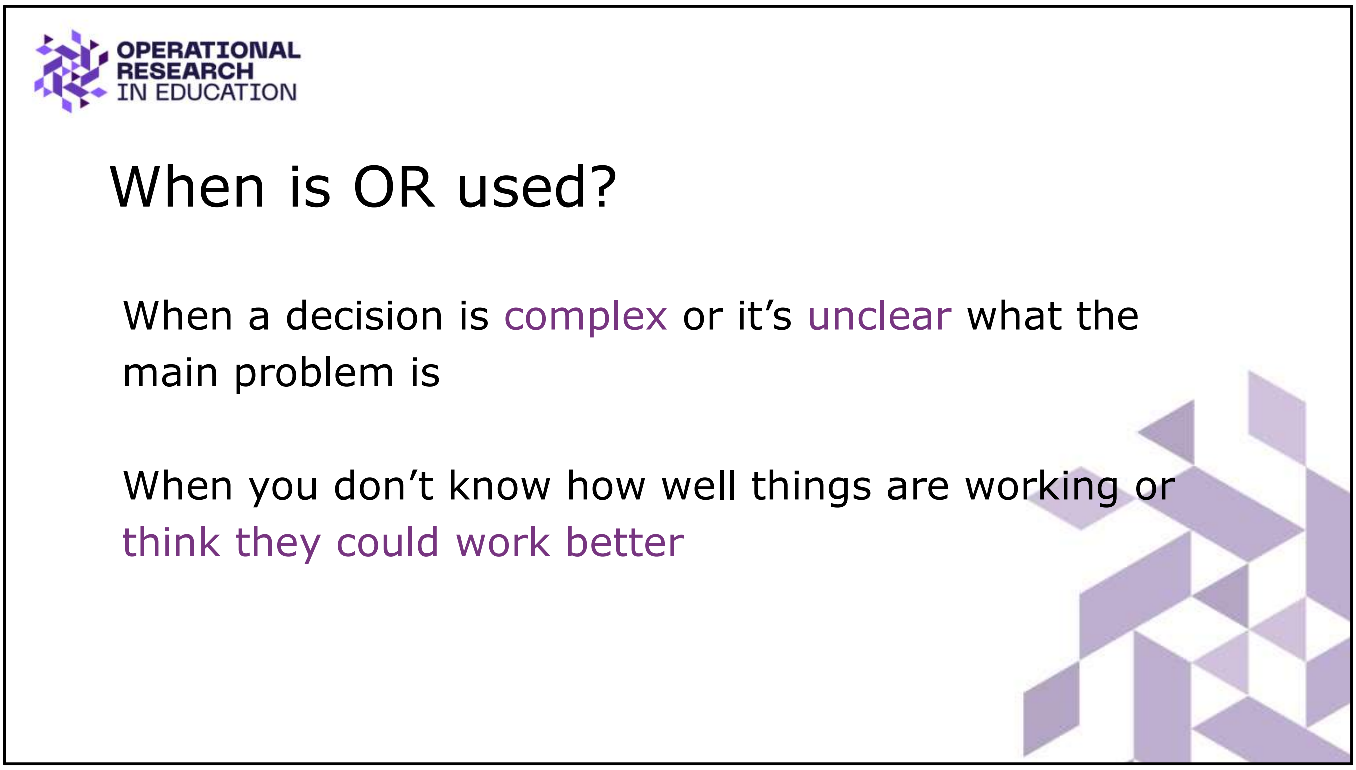
Please feel free to paraphrase the below:

Some hospitals have dedicated OR teams to help with resource allocation – especially if they have multiple specialities. The OR staff allocate patients, equipment and surgical teams to operating theatres based on the urgency and specific requirements of each patient – some operations need specialist equipment and others do not and it's not very efficient to have a 'general' patient in a 'specialist' surgery.

The OR team have to set a schedule, which is made complicated by the fact that how long an operation takes can be hard to predict and an emergency patient might need immediate attention and throw off the rest of the rota!

OR researchers designed an algorithm to optimise kidney transplant surgery – imagine somebody needs a kidney transplant and their family member is willing to be a donor, but is incompatible. The algorithm identifies patients in this situation and matches them up so they can swap donors, and both patients receive the kidney that they need.

The surgery has to take place simultaneously to prevent anybody from backing out at the last minute, so the algorithm also has to take into account the nearest hospital with enough resources (theatres and surgical teams) to carry out the transplant when matching patients.



Decision-making and problem-solving in business can be complicated and messy. It may not be clear what the main problem is, what the outcome of different actions may be or how well things are currently working, and there may be lots of different factors to consider.

For example, if things don't go well when businesses make big changes, they might upset customers, slow down production or create a need for extra staff training. Any of these could have a negative impact on the business. OR can help to reduce the chances of this happening.

OPERATIONAL RESEARCH IN EDUCATION

What OR techniques are used?

Simulation is used to try different solutions and answers the "What if...?" question.

Optimisation uses problem solving to achieve the best outcome.

Forecasting is used to estimate unknown outcomes with more accuracy.

Some commonly used OR techniques include:

Optimisation – depending on what variable is most important (manufacturing something quickly, or maximising profit?), optimisation will find the best use of limited resources.

Simulation – this modelling tool is fantastic when there are a lot of different ways to solve a problem as you can try lots of different solutions until you find the best one. It also allows something to be tested in a safe way, for example, organisations like the NHS have to be careful when making changes as lives could be at risk!

Forecasting – forecasting can be used to try and predict unknown factors, to help keep a business running smoothly. For example, estimating customer demand so companies know which goods to produce or forecasting the impact of rush hour traffic on a delivery route, so the driver can stay on schedule.

Also many more techniques – including algorithms!

Non-exhaustive list of businesses that use OR. Please note they are not endorsed by the OR Society but are designed to show the variety of careers in OR. Feel free to use your own examples if relevant.

Pause for questions

OPERATIONAL RESEARCH IN EDUCATION

Interested?

Next steps:

- Maths GCSE and A Level
- Further Maths and Computer Science are highly beneficial
- Obtain a good classification in a STEM degree
- A masters degree in OR is often desirable, although not always essential

Depending on the age of the audience this may or may not be relevant.

Not many universities offer OR degrees, although some offer maths and OR degrees or similar. OR is often a module in a maths or business studies degree and can be hard to find on its own.

STEM degrees (science, technology, engineering and maths) show a skill set and analytical way of thinking that is often beneficial to people working in OR and are a good alternative to an (often elusive) OR degree.

OPERATIONAL RESEARCH IN EDUCATION

Find out more

www.theorsociety.com

Careers information, OR news and information on free student membership (available for people aged 16+) can be found on our website.